Evaluating the Trade-Offs in Tax and Transfer Policy: Theory and Danish Evidence

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Contents

1. Introduction

Kleven and Kreiner (2006a,b,c,d) are part of a larger research project by the Rockwool Foundation Research Unit on work and the welfare state. Kleven and Kreiner (2006c,d) evaluate the effects of the Danish tax and transfer policy on labor supply incentives, government revenue, economic efficiency and equality. This paper provides a technical documentation of the economic theory and numerical analysis underlying the results.

Part I describes the modelling of labor supply and derives theoretical measures of the effects of tax reforms on government revenue, the marginal cost of public funds and the trade-off between efficiency and equality. Part I also derives conditions under which tax cuts pay for themselves. The key theoretical measures are generalizations of existing results in the public finance literature, see Immervoll *et al.* (2005), Kleven and Kreiner (2006e) and Saez (2004).

The theory is applied to data from the Danish Law Model. Part II describes the data and the methodology behind the numerical implementation.

Part I

Economic Theory

2. Modelling of Labor Supply Behavior

In this section, we present the model of labor supply underlying the analysis. As we have explained elsewhere (e.g. Kleven and Kreiner, 2005), it is very important to distinguish explicitly between labor supply adjustments along the intensive margin (the number of hours worked) and along the extensive margin (the number of employed individuals). Moreover, a realistic model must be consistent with empirical distributions of hours worked showing almost no workers a low weekly hours of work. In other words, if individuals decide to work at all, they tend to work a substantial number of hours (say, 30 or 40 hours per week). In the empirical literature, discrete entry is typically explained by the presence of non-convexities in preferences and/or budget sets created by fixed work costs (e.g., Cogan, 1981) or concave work cost functions (Heim and Meyer, 2003). These work costs may be monetary costs (say, expenses to child care and transportation), they may reflect time losses (say, commuting time), or they could be emotional costs arising from the added responsibility and stress associated with having a job. Work costs of this sort — whether they are fixed or depend in some way on working hours — tend to create economies of scale in the work decision making very low hours of work unattractive. Below we adopt a framework incorporating fixed costs of working, denoted by q, in order to get discrete entry. We allow for heterogeneity in the size of q, along with heterogeneity in wages/productivities and preferences.

We assume that the population may be divided into I distinct subgroups, with N_i denoting the number of individuals in group i. Across different subgroups, we allow for heterogeneity in wages rates as well as in preferences. Within any given group, individuals are characterized by identical wages and preferences, but they are facing heterogeneous fixed costs of work. In particular, we assume a continuum of individuals in each group, with fixed work costs in group i being distributed according to the cumulative distribution function $P_i(q)$ and the density function $p_i(q)$. By assuming a continuum of fixed costs, the model will generate a smooth participation response at the aggregate level of the group, such that the sensitivity of entry-and-exit behavior may be captured by elasticity parameters for each group. Although the aggregate participation response in each group is continuous in this way, the individual participation response is discrete as explained in the previous section.

Individual utility is specified as

$$v_i(c, h) - q \cdot 1(h > 0), \tag{2.1}$$

where c is consumption, h is hours of work, and $1(\cdot)$ denotes the indicator function. The subutility function $v_i(c, h)$ is increasing in the first argument and decreasing in the second argument. While the subutility function $v_i(\cdot)$ is assumed to be well-behaved, a non-convexity is introduced through the last component in utility. Conditional on labor market participation ($h > 0$), the individual incurs a fixed utility cost q along with the standard variable disutility of work embodied in the v_i-function. The specification implies that the average work cost per hour is U-shaped – like a standard average total cost curve in production theory –

and this tends to make small hours of work unattractive for the individual. If the individual enters the labor market at all, he would do so at some minimum number of working hours, say 20 or 30 per week.

For the purpose of simplification, we have assumed separability of fixed costs in the utility function (2.1). Because of this assumption, the size of the fixed cost will not affect the marginal rate of substitution between leisure and consumption conditional on entry. Hence, while the fixed cost is crucial for the decision to enter the labor market, it will not affect the choice of hours worked once entry has been made. In combination with identical within-group wage rates and preferences, the separability of fixed costs implies that every individual in a given group enters the labor market at the same hours of work. In other words, while the entry-and-exit decision is heterogeneous within the group – due to different work costs – decisions about hours of work and earnings conditional on entry are not. As we shall see below, this formulation allows us to capture labor supply responses by setting intensive and extensive elasticities at the level of the group, and it allows us to capture the tax-transfer system by setting marginal tax rates along with virtual incomes for each group rather than each individual.

A more general formulation would include q as a third argument in the v_i-function. This implies a correlation of fixed costs with earnings and intensive labor supply responses within each group. Although this extension would make the analysis a lot more involved, the general formulation is analytically tractable. But for the purpose of empirical application, it is unclear that much would be gained by the added complexity. In the end, fixed costs are difficult to observe and thus little is is known about their correlation with earnings and labor supply responses. Moreover, the error made by ignoring a possible correlation between fixed costs and labor supply behavior at the intensive margin may be alleviated by disaggregating groups more. If we were to consider highly disaggregated groups – say income deciles along with demographic subgroups at each decile – the assumption of uniform within-group behavior at the intensive margin seems quite reasonable.

Taxes and transfers in this model are captured by a net-tax payment function $T(w_i h, \mathbf{x}_i, z)$, where w_i denotes an exogenous gross wage rate, and where z is a shift-parameter that we use to capture changes in the tax-transfer system. Besides earnings, the tax-transfer scheme may depend upon non-labor income and demographic characteristics (kids, marital status, place of residence, etc.) included in the vector $\mathbf{x}_i$. While we allow for the T-function to include non-linearities and discontinuities, we restrict attention to the case of piecewise linearity. The budget constraint for individuals in group i equals

$$c \leq w_i h - T(w_i h, \mathbf{x}_i, z), \tag{2.2}$$

which may alternatively be written as

$$c \leq (1 - m_i) w_i h + Y_i, \tag{2.3}$$

where $m_i \equiv \partial T(w_i h, \mathbf{x}_i, z) / \partial (w_i h)$ is the marginal tax rate and $Y_i \equiv m_i w_i h - T(w_i h, \mathbf{x}_i, z)$ is so-called virtual income.

The household maximizes (2.1) subject to (2.3). The problem is solved in a two-step procedure: first, we solve for the optimal hours of work conditional on working, and then we consider the choice to participate in the labor market. Conditional on participation ($h > 0$), the optimum is characterized by the standard first-order condition

$$-\frac{\partial v_i(c_i, h_i) / \partial h}{\partial v_i(c_i, h_i) / \partial c} = (1 - m_i) w_i, \tag{2.4}$$

where c_i and h_i denote optimal consumption and hours worked for a participating worker in group i. From (2.3) and (2.4), we may write hours worked as a function of the marginal net-of-tax wage rate and the virtual income, i.e. $h_i = h_i \left(\left(1 - m_i \right) w_i, Y_i \right)$.

For the individual to enter the labor market in the first place, the utility from participation must be greater than or equal to the utility from non-participation. This implies an upper bound on the fixed cost of working:

$$q \leq v_i \left(c_i, h_i \right) - v_i \left(c_i^0, 0 \right) \equiv \bar{q}_i, \tag{2.5}$$

where $c_i^0 \equiv -T \left(0, \mathbf{x}_i, z \right)$ denotes the consumption level for non-participants in group i. Individuals with a fixed cost below the threshold-value $\bar{q}_i$ enter the labor market at h_i hours, while those with a fixed cost above the threshold decide not to enter. Since the fixed cost is distributed according to the density function $p_i \left(q \right)$, the fraction of individuals in group i who participate in the labor market is given by $\int_0^{\bar{q}_i} p_i \left(q \right) dq = P_i \left(\bar{q}_i \right)$. The aggregate labor supply in group i is a product of the number of individuals participating in the labor market and the hours of work for these individuals, i.e.

$$L_i = N_i \cdot P_i \left(\bar{q}_i \right) \cdot h_i \left(\left(1 - m_i \right) w_i, Y_i \right). \tag{2.6}$$

This expression decomposes aggregate labor supply into labor supply along the extensive and intensive margins, and it implies that variation in aggregate labor supply reflects changes along these two margins. The explicit distinction between margins of response is necessary due to the fact that the two margins are related to distinct tax-transfer parameters. While the choice of working hours depends on the marginal tax rate m_i, the participation rate is determined by the threshold-value for the fixed cost $\bar{q}_i$, which is related to total tax liabilities when working and not working, $T(w_i h_i, \mathbf{x}_i, z)$ and $T(0, \mathbf{x}_i, z)$.

The sensitivity of labor supply along each margin is measured by labor supply elasticities. The hours-of-work elasticity with respect to the after-tax wage rate is defined as

$$\varepsilon_i \equiv \frac{\partial h_i}{\partial\left[(1-m_i) w_i\right]} \frac{(1-m_i) w_i}{h_i}, \tag{2.7}$$

which denotes the uncompensated hours-of-work elasticity. From the Slutsky-equation, it may be decomposed into a compensated elasticity, ε_i^c, and an income effect, θ_i, that is

$$\varepsilon_i = \varepsilon_i^c - \theta_i \gtrless 0, \tag{2.8}$$

where $\theta_i \equiv -(1-m_i) w_i \cdot \partial h_i / \partial Y_i$ is positive if leisure is a normal good. The hours-of-work elasticity has an indeterminate sign reflecting that the individual labor supply curve may be backward-bending.

The sensitivity along the extensive margin is measured by two participation elasticities:

$$\eta_i \equiv \frac{c_i - c_i^0}{P_i} \frac{\partial P_i}{\partial (c_i - c_i^0)} \frac{\partial (c_i - c_i^0)}{\partial c_i} = \frac{(c_i - c_i^0) p_i(q_i)}{P_i(q_i)} \frac{\partial v_i(c_i, h_i)}{\partial c_i}, \tag{2.9}$$

$$\eta_i^0 \equiv -\frac{c_i - c_i^0}{P_i} \frac{\partial P_i}{\partial (c_i - c_i^0)} \frac{\partial (c_i - c_i^0)}{\partial c_i^0} = \frac{(c_i - c_i^0) p_i(q_i)}{P_i(q_i)} \frac{\partial v_i(c_i^0, 0)}{\partial c_i^0}. \tag{2.10}$$

Both elasticities measure the percentage change in the number of workers in group i following a one-percent increase in the difference in consumption between working and not working, $c_i - c_i^0$. The change in the incentive to participate may occur through a change in the budget set at the working state, captured by η_i, and/or a change at the non-working state, captured by η_i^0. The two elasticities are not necessarily identical because the marginal utility of consumption may differ between the two states. Unless leisure is an inferior good, the marginal utility of consumption is higher at the non-working state than at the working state, $\frac{v_i\left(c_i^0,0\right)}{\partial c_i^0} \geq \frac{\partial v_i(c_i,h_i)}{\partial c_i}$. This implies that $\eta_i^0 \geq \eta_i$. The difference between the two elasticities reflects an income effect in the participation decision making. If income increases with the same amount in the two states, for example because of an increase in non-labor income or because of a general tax reduction, then labor force participation will fall if $\eta_i^0 > \eta_i$.

Notice that the participation elasticities are positive. By implication, a rise in the wage rate always increases labor supply along the extensive margin. This is in contrast to labor supply responses along the intensive margin which may be negative due to a strong income effect on hours worked.

3. Effects of Tax Reforms on Government Revenue

In this section, we analyze the impact of tax reforms on government revenue. The aggregate government revenue is given by

$$R = \sum_{i=1}^{I} \left[T\left(w_i h_i, \mathbf{x}_i, z\right) P_i\left(\bar{q}_i\right) + T\left(0, \mathbf{x}_i, z\right)\left(1 - P_i\left(\bar{q}_i\right)\right) \right] N_i, \qquad (3.1)$$

where the first term reflects total tax payments net of transfers for those who are working, while the second term is total tax payments net of transfers (presumably a negative number) for those outside the labor market. A tax reform, represented by a change in the shift parameter z, influences government revenue through a mechanical effect and a behavioral effect:

$$\frac{dR}{dz} = \frac{dM}{dz} + \frac{dB}{dz}, \qquad (3.2)$$

$$\frac{dM}{dz} = \sum_{i=1}^{I} \left[\frac{\partial T_i}{\partial z} P_i\left(\bar{q}_i\right) + \frac{\partial T_i^0}{\partial z}\left(1 - P_i\left(\bar{q}_i\right)\right) \right] N_i, \qquad (3.3)$$

$$\frac{dB}{dz} = \sum_{i=1}^{I} \left[m_i w_i \frac{dh_i}{dz} P_i\left(\bar{q}_i\right) + \left(T_i - T_i^0\right) \frac{dP_i\left(\bar{q}_i\right)}{dz} \right] N_i, \qquad (3.4)$$

where we have used the simplifying notation $T_i \equiv T\left(w_i h_i, \mathbf{x}_i, z\right)$ and $T_i^0 \equiv T\left(0, \mathbf{x}_i, z\right)$. The mechanical effect in (3.3) measures the direct impact of the reform on government revenue before any behavioral responses. The behavioral feed-back effects on revenue are given in (3.4). The first term shows the revenue effect created by adjustments in hours worked by those who are working. The size of this revenue effect depends on the magnitudes of the initial marginal tax rates. The second term reflects that some workers will be induced to join the ranks of non-employed people when the tax burden on labor income goes up, creating a revenue effect because of lower tax proceeds and higher aggregate transfer payments to those out of work. The expression shows that the size of this revenue effect depends on the initial tax burden on labor force participation, $T_i - T_i^0$.

Using eqs (2.2) to (2.10), the expression in (3.2) may be rewritten to (see Appendix A)

$$\begin{aligned} \frac{dR}{dz} &= \sum_{i=1}^{I} \left[\frac{\partial T_i}{\partial z} E_i + \frac{\partial T_i^0}{\partial z} (N_i - E_i) - \frac{m_i}{1-m_i} \left(\frac{\partial m_i}{\partial z} w_i h_i \varepsilon_i^c - \frac{\partial T_i}{\partial z} \theta_i \right) E_i \right. \\ &\quad \left. - \frac{a_i + b_i}{1 - a_i - b_i} \left(\frac{\partial T_i}{\partial z} \eta_i - \frac{\partial T_i^0}{\partial z} \eta_i^0 \right) E_i \right], \end{aligned} \tag{3.5}$$

where $E_i = P_i(\bar{q}_i) N_i$ is the number of employed individuals in group i, $b_i \equiv -T_i^0/(w_i h_i)$ denotes a benefit rate, while $a_i \equiv T_i/(w_i h_i)$ is the average tax rate.

Let $\frac{\partial a_i}{\partial z} \equiv \frac{\partial T_i}{\partial z}/(w_i h_i)$ and $\frac{\partial b_i}{\partial z} \equiv \frac{\partial T_i^0}{\partial z}/(w_i h_i)$ be the change in the average tax rate and the benefit rate, respectively, following the reform (excluding behavioral responses). Then the above expression may be rewritten to

$$\begin{aligned} \frac{dR}{dz} &= \sum_{i=1}^{I} \left[\frac{\partial a_i}{\partial z} - \frac{\partial b_i}{\partial z} \frac{N_i - E_i}{E_i} - \frac{m_i}{1-m_i} \left(\frac{\partial m_i}{\partial z} \varepsilon_i^c - \frac{\partial a_i}{\partial z} \theta_i \right) \right. \\ &\quad \left. - \frac{a_i + b_i}{1 - a_i - b_i} \left(\frac{\partial a_i}{\partial z} \eta_i + \frac{\partial b_i}{\partial z} \eta_i^0 \right) \right] w_i L_i, \end{aligned} \tag{3.6}$$

where $\frac{\partial a_i}{\partial z} \equiv \frac{\partial T_i}{\partial z}/(w_i h_i)$ and $\frac{\partial b_i}{\partial z} \equiv \frac{\partial T_i^0}{\partial z}/(w_i h_i)$ is the change in the average tax rate and the benefit rate, respectively, following the reform (excluding behavioral responses). This expression decomposes the intensive revenue effect into a substitution effect being created by the change in the marginal tax rate and an income effect resulting from the change in the average tax rate. We also see that the extensive response is being created by the change in the average tax rate and the benefit rate, and that the revenue implications of these responses have to do with the size of $a_i + b_i$, which is the total *participation tax rate.*

Of course, big tax reforms have, ceteris paribus, a large impact on government revenue. When comparing different types of reforms, it is therefore necessary to normalize the size of the reforms. We do so by deriving the behavioral effect on revenue per dollar raised in mechanical revenue, i.e.,

$$S \equiv -\frac{dB/dz}{dM/dz} = -\frac{dR/dz - dM/dz}{dM/dz}, \tag{3.7}$$

which gives the fraction of tax revenue lost through behavioral responses. In the case of tax cuts, S measures how big a fraction of the tax cut that is recouped by behavioral feed-back effects on government revenue. Hence, we will refer to S as the degree of self-financing.

In the following, we will consider different types of tax reforms. For that purpose, it is useful to derive a general formula for tax reforms that keep the well-being of the unemployed unchanged, $\partial T_i^0/\partial z = \partial b_i/\partial z = 0$. By inserting (3.6) in the definition of S, we obtain

$$S = \sum_{i=1}^{I} \left[\frac{m_i}{1-m_i} \left(\Phi_i \varepsilon_i^c - \theta_i \right) + \frac{a_i + b_i}{1 - a_i - b_i} \eta_i \right] s_i, \tag{3.8}$$

where $\Phi_i \equiv \frac{\partial m_i/\partial z}{\partial a_i/\partial z}$ is a measure of the progressivity of the tax change, and where $s_i \equiv \frac{\partial a_i}{\partial z} w_i L_i / \left(\sum_{i=1}^{I} \frac{\partial a_i}{\partial z} w_i L_i \right)$ denotes the tax increase for group i as a share of the total tax increase in the population.

3.1. A Lump Sum Tax Change for All Individuals (Reform A)

We first consider a tax reform that changes the tax liabilities for all employed, $\partial T_i/\partial z$, and all unemployed, $\partial T_i^0/\partial z$, with the same amount. Notice, that this reform leaves the marginal incentive to supply hours unaffected because the marginal tax rate is unchanged. The difference in consumption possibilities between working and not working is also unchanged. Hence, the change in labor supply behavior is only due to income effects.

From eqs (3.5) and (3.7), we get

$$S^A = \rho \sum_{i=1}^{I} \left[-\frac{m_i}{1-m_i}\theta_i - \frac{a_i+b_i}{1-a_i-b_i}\left(\eta_i^0 - \eta_i\right) \right] e_i, \tag{3.9}$$

where $\rho \equiv E/N$ is the participation rate and where $e_i \equiv E_i/E$ is the employment share of group i. The reform gives rise to two types of income effects on labor supply behavior, reflected by the two terms in the expression. Firstly, the higher tax burden on employed individuals creates an income effect leading to higher hours of work (under the assumption that leisure is a normal good). The implied increase in earnings is taxed with the marginal tax rate and this raises tax revenue, as reflected by the first term in the above expression (S^A becomes negative reflecting that a lump sum tax reduction will generate reductions in labor supply which will reduce tax revenue further).

The second term reflects an income effect in the participation decision. The tax burden is raised with the same amount on employed and unemployed individuals. However, the marginal utility of income may be higher in the unemployment state than in the employment state, leading to $\eta_i^0 > \eta_i$, in which case some individuals will be induced to enter the labor market. These individuals leave welfare benefits and pay more in taxes thereby improving the government budget.

3.2. A Lump Sum Tax Change for Employed Individuals (Reform B)

In this case, the tax burden is raised with the same amount on all employed individuals ($\partial T_i/\partial z$ is the same for all i) while the net-tax payment of the unemployed is left unchanged ($\partial T_i^0/\partial z = 0$). Thus, conditional on labor force participation, the increase in tax burden amounts to a lump sum payment.

By inserting $\Phi_i = 0$ in expression (3.8) and noting that s_i for this reform is simply equal to the employment share, we obtain

$$S^B = \sum_{i=1}^{I} \left[-\frac{m_i}{1-m_i}\theta_i + \frac{a_i + b_i}{1-a_i-b_i}\eta_i \right] e_i. \tag{3.10}$$

Compared to (3.9) only the second term differs. The reason for this difference is that the reform in this case only raises tax liabilities on employed. This will unambiguously reduce labor market participation and thereby government revenue.

3.3. A Lump Sum Tax Change for All Individuals with Certain Demographic Characteristics (Reform C)

In this case, the tax burden is raised with the same amount on all individuals who belong to certain demographic groups (tagging). Individuals who are target for the reform pay the same lump sum tax. Let Υ denote the groups who are target for the reform. From eqs (3.5) and (3.7), we obtain

$$S^C = \rho \sum_{\Upsilon} \left[-\frac{m_i}{1-m_i}\theta_i - \frac{a_i + b_i}{1-a_i-b_i}\left(\eta_i^0 - \eta_i\right) \right] e_i, \tag{3.11}$$

where $e_i \equiv E_i / \sum_{\Upsilon} E_i$ and $\rho \equiv \frac{\sum_{\Upsilon} E_i}{\sum_{\Upsilon} N_i}$. Notice, that this result is identical to S^A when all individuals are given transfers.

3.4. A Lump Sum Tax Change for Employed Individuals with Certain Demographic Characteristics (Reform D)

In this case, the tax burden is raised with the same amount on all employed individuals who belong to certain demographic groups. Individuals who are target for the reform pay the same lump sum tax. Let Υ denote the groups who are target for the reform. Then we obtain

$$S^D = \sum_{\Upsilon} \left[-\frac{m_i}{1-m_i}\theta_i + \frac{a_i + b_i}{1 - a_i - b_i}\eta_i \right] e_i, \tag{3.12}$$

which is identical to S^B if all groups are target for the reform. On the other hand, a reform targeted at individuals with high participation elasticities and high participation tax rates may involve a substantial higher degree of self-financing.

3.5. A Proportional Tax Change (Reform E)

A natural and simple benchmark case to study is where additional revenue is raised through a proportional tax change. In this case, the marginal tax rate is increased at all income levels and with the same percentage point increase at all income levels, i.e., $\partial m_i / \partial z = \partial m_j / \partial z$ for all i, j. In this case, the change in the average tax rate $\partial a_i / \partial z$ is equal to the change in the marginal tax rate $\partial m_i / \partial z$, which implies that expression (3.8) simplifies to

$$S^E = \sum_{i=1}^{I} \left[\frac{m_i}{1-m_i}\varepsilon_i + \frac{a_i + b_i}{1 - a_i - b_i}\eta_i \right] s_i, \tag{3.13}$$

where $\varepsilon_i = \varepsilon_i^c - \theta_i$ is the uncompensated hours-of-work elasticity, and where $s_i = w_i L_i / \left(\sum_{i=1}^{I} w_i L_i \right)$ is the earnings share of group i.

3.6. A Marginal Tax Increase at a Given Earnings Level (Reform F)

Another interesting special case is where additional government revenue is collected by increasing the marginal tax rate at one earnings level, holding all other marginal tax rates constant. For the purpose of studying this case, we impose an ordering of subgroups $i = 1, ..., I$ according to earnings levels such that the earnings level in group i is higher than in group $i - 1$. Then we consider an increase in the marginal tax rate in group k, while all other marginal tax rates are unchanged. In this case, eq. (3.8) may be rewritten to

$$S_{\mathrm{k}}^F = \frac{m_k}{1 - m_k} \Phi_k \varepsilon_k^c s_k - \sum_{i=k}^{I} \frac{m_i}{1 - m_i} \theta_i s_i + \sum_{i=k}^{I} \frac{a_i + b_i}{1 - a_i - b_i} \eta_i s_i \tag{3.14}$$

where we have used the fact that groups below k are unaffected by the reform. In the empirical implementation, we will also account for heterogeneity within earnings groups, which may lead to different tax rates and elasticities. However, to understand the main mechanisms, it is useful to start by considering the above more simple case.

The reform gives rise to three different effects on labor supply behavior, reflected by the three terms in the expression. Firstly, because of the higher marginal tax rate for people in group k, workers in this group reduce hours worked through a substitution effect. This effect reduces government revenue. Secondly, all workers with earnings at – or above – the level of group k experience higher tax burdens, which creates an income effect leading to higher hours of work if leisure is a normal good. In isolation, this effect implies a higher revenue. Finally, increased tax burdens will induce some workers to exit the labor market in order to collect benefits, thereby lowering government revenue.

An interesting issue is the relation between the degree of self-financing and the earnings level where marginal tax rates are being increased, i.e., how S depends on the earnings threshold of the reform k. It turns out that there are many different effects at play and that it is impossible from theory alone to determine whether S increases or decreases when k is changed. The theoretical effects are discussed in greater detail in Kleven and Kreiner (2006e). In Part II, we will calibrate the degree of self-financing for Denmark and study how it is related to the earnings threshold of the reform.

In the empirical implementation, we will divide the population into earnings deciles. Thus, i denotes the decile number and I equals 100. We will account for the fact that effective tax rates may vary because tax liabilities and benefit payments depend on age, marital status, kids, etc. In addition, labor supply elasticities may vary across demographic sub-groups. Let J be the relevant number of combinations of demographic characteristics. The revised version of (3.14) then becomes

$$
\begin{aligned}
S_{\mathrm{k}}^{F} &= \Phi_k s_k A_k - \sum_{i=k}^{100} s_i B_i + \sum_{i=k}^{100} s_i C_i, \qquad (3.15)\\
A_i &\equiv \sum_{j=1}^{J} \frac{m_{ij}}{1-m_{ij}} \varepsilon_{ij}^{c} \frac{E_{ij}}{E_i},\\
B_i &\equiv \sum_{j=1}^{J} \frac{m_{ij}}{1-m_{ij}} \theta_{ij} \frac{E_{ij}}{E_i},\\
C_i &\equiv \sum_{j=1}^{J} \frac{a_{ij}+b_{ij}}{1-a_{ij}-b_{ij}} \eta_{ij} \frac{E_{ij}}{E_i},
\end{aligned}
$$

where E_{ij}/E_i denotes the number of individuals in group (i, j) in proportion to all individuals in decile i. As in the previous sections, the variable s_i denotes the tax increase for group i as a share of the total tax increase in the population. Notice, however, that the tax increase for group i depends on the decile k at which the marginal tax rate is raised. Indeed, we have

$$
s_i = \begin{cases} 0 & \text{if } i < k \\ \frac{w_k h_k - y_k}{w_k h_k - y_k + (I-k)(y_{k+1}-y_k)} & \text{if } i = k \\ \frac{y_{k+1}-y_k}{w_k h_k - y_k + (I-k)(y_{k+1}-y_k)} & \text{if } i > k \end{cases}, \qquad (3.16)
$$

where y_k is the earnings level that separates percentile k and $k-1$. Here, $w_k h_k - y_k$ is the income in percentile k that is subject to the higher tax rate, and $y_{k+1} - y_k$ is the income that is subject to the higher tax rate for individuals belonging to percentiles higher than k (recall that the marginal tax rate is raised for earnings in percentile k, i.e. in the earnings interval $y_{k+1} - y_k$). Since the number of individuals are constant across percentiles, $\frac{w_k h_k - y_k}{w_k h_k - y_k + (I-k)(y_{k+1}-y_k)}$ measures the fraction of the tax increase paid by individuals in percentile k, while $\frac{y_{k+1}-y_k}{w_k h_k - y_k + (I-k)(y_{k+1}-y_k)}$ measures the fraction of the tax increase paid by individuals in a given percentile above k.

The increase in tax burden of an individual in percentile k equals $\partial m_k/\partial z \cdot (w_k h_k - y_k)$. Therefore, the change in the average tax rate becomes $\partial a_k/\partial z = \partial m_k/\partial z \cdot (w_k h_k - y_k) \,/\, (w_k h_k)$ implying that

$$\Phi_k = \frac{\partial m_k/\partial z}{\partial a_k/\partial z} = \frac{w_k h_k}{w_k h_k - y_k}. \tag{3.17}$$

From these formulas and information on earnings, effective tax rates and labor supply elasticities, we are able to calculate the degree of self-financing for every k.

3.7. A Marginal Tax Increase on All Income Above a Given Threshold (Reform G)

For this reform, we assume that the marginal tax rates are increased on all labor income above y_k where k denotes an earnings percentile while y_k is the earnings level that separates percentile k and $k-1$. Thus, the marginal tax rates are raised for all the groups $i \geq k$, while marginal taxes are unchanged for the groups $i < k$. It is assumed that the marginal tax rate increase, $\partial m_i/\partial z$, is the same for all the groups $i \geq k$.

In this case, eq. (3.8) may be rewritten to

$$
\begin{aligned}
S_{\mathrm{k}}^{G} &= \sum_{i=k}^{100} \Phi_i s_i A_i - \sum_{i=k}^{100} s_i B_i + \sum_{i=k}^{100} s_i C_i \qquad (3.18)\\
A_i &\equiv \sum_{j=1}^{J} \frac{m_{ij}}{1-m_{ij}} \varepsilon_{ij}^{c} \frac{E_{ij}}{E_i},\\
B_i &\equiv \sum_{j=1}^{J} \frac{m_{ij}}{1-m_{ij}} \theta_{ij} \frac{E_{ij}}{E_i},\\
C_i &\equiv \sum_{j=1}^{J} \frac{a_{ij}+b_{ij}}{1-a_{ij}-b_{ij}} \eta_{ij} \frac{E_{ij}}{E_i}.
\end{aligned}
$$

This result has some similarity to the result (3.15) for reform F. The variables A_i, B_i and C_i are the same as before, and so are the last two terms in (3.18). The first term in (3.18) is different because all groups above k now face higher marginal tax rates which lead to substitution effects in the hours-of-work decisions. A more subtle difference is taking place in the s_i-parameters:

$$
s_i = \frac{w_i h_i - y_k}{\sum_{i=k}^{100} (w_i h_i - y_k)} \text{ for } i \geq k. \qquad (3.19)
$$

These parameters differ from the previous reform because all income above y_k now is taxed at a higher rate. In reform F, the marginal tax rate was only raised in the income interval $y_{k+1} - y_k$.

Finally, we have

$$
\Phi_i = \frac{\partial m_i / \partial z}{\partial a_i / \partial z} = \frac{w_i h_i}{w_i h_i - y_k} \text{ for } i \geq k,
$$

which is now defined for all $i \geq k$ because marginal tax rates are raised on all income above y_k.

In Part II, we describe how these expressions may be applied to data in order to study the consequences of the current tax and welfare system. The results from a numerical implementation on Danish data are reported in Kleven and Kreiner (2006c).

3.8. Budget-Neutral Tax Reforms

In this section, we consider tax reforms where the proceeds from a tax increase are returned back to all individuals as a lump-sum transfer, TR. The size of the lump sum grant is adjusted to the level that ensures (ex post) revenue neutrality, i.e. $dR/dz = 0$. Consider for example the proportional tax change studied in Section 3.5. The mechanical revenue effect is denoted by $\left|\frac{dM}{dz}\right|_E$ implying that the total effect on government revenue equals $\left|\frac{dM}{dz}\right|_E (1 - S_E)$. We may now combine this reform with the lump-sum reform studied in Section 3.1 in order to get

$$\left|\frac{dM}{dz}\right|_E \cdot (1 - S_E) = TR \cdot N \cdot (1 - S_A).$$

This may be rewritten as

$$\left|\frac{dM}{dz}\right|_E \cdot (1 - S_{\tilde{E}}) = TR \cdot N,$$

where $S_{\tilde{E}} = 1 - \frac{1-S_E}{1-S_A}$ is the degree of self-financing of Reform E including budget-neutral lump-sum transfers. The other reforms studied in the preceding sections may be adjusted in the same way in order to ensure budget neutrality through lump sum transfers.

3.A. Derivation of equation (3.5)

In this appendix, we derive eq. (3.5). This is done by first deriving the behavioral responses to the tax reform, dh_i/dz and $dP_i\left(\bar{q}_i\right)/dz$, and then inserting the results into eq. (3.2). We start by deriving $dP_i(\bar{q}_i)/dz$. From the participation condition (2.5), we have

$$\frac{d\bar{q}_i}{dz} = \frac{\partial v_i(c_i, h_i)}{\partial c_i}\frac{dc_i}{dz} + \frac{\partial v_i(c_i, h_i)}{\partial h_i}\frac{dh_i}{dz} - \frac{\partial v_i(c_i^0, 0)}{\partial c_i^0}\frac{dc_i^0}{dz}.$$

Using the budget constraint (2.2) to derive dc_i/dz, the above relationship may be written as

$$\begin{aligned}\frac{d\bar{q}_i}{dz} &= \left[\frac{\partial v_i(c_i, h_i)}{\partial c_i}(1-m_i)w_i + \frac{\partial v_i(c_i, h_i)}{\partial h_i}\right]\frac{dh_i}{dz} \\ &\quad -\frac{\partial v_i(c_i, h_i)}{\partial c_i}\frac{\partial T_i}{\partial z} + \frac{\partial v_i(c_i^0, 0)}{\partial c_i^0}\frac{\partial T_i^0}{\partial z}.\end{aligned}$$

From the first-order condition for hours of work (eq. 2.4), the first term in this expression is equal to zero, such that we obtain

$$\frac{d\bar{q}_i}{dz} = -\frac{\partial v_i(c_i, h_i)}{\partial c_i}\frac{\partial T_i}{\partial z} + \frac{\partial v_i(c_i^0, 0)}{\partial c_i^0}\frac{\partial T_i^0}{\partial z}.$$

This expression implies that the change in the participation rate is equal to

$$\begin{aligned}\frac{dP_i(\bar{q}_i)}{dz} &= p_i(\bar{q}_i)\frac{d\bar{q}_i}{dz} \\ &= -p_i(\bar{q}_i)\left(\frac{\partial v_i(c_i, h_i)}{\partial c_i}\frac{\partial T_i}{\partial z} - \frac{\partial v_i(c_i^0, 0)}{\partial c_i^0}\frac{\partial T_i^0}{\partial z}\right). \qquad (3.20)\end{aligned}$$

In this equation, we substitute the participation elasticities defined in (2.10) and (2.9). This gives the following change in the participation rate

$$\frac{dP_i(\bar{q}_i)}{dz} = -\frac{1}{c_i - c_i^0}\left[\eta_i\frac{\partial T_i}{\partial z} - \eta_i^0\frac{\partial T_i^0}{\partial z}\right]P_i(q_i) \qquad (3.21)$$

Next, we derive the impact on hours worked of a small reform. The optimal hours of work is a function of the marginal net-of-tax wage rate and the virtual income, i.e. $h_i = h_i((1-m_i)w_i, Y_i)$. A reform changes hours according to

$$\frac{dh_i}{dz} = -\frac{\partial h_i}{\partial[(1-m_i)w_i]}w_i\frac{\partial m_i}{\partial z} + \frac{\partial h_i}{\partial Y_i}\frac{dY_i}{dz}.$$

The definition of virtual income, $Y_i \equiv m_i w_i h - T(w_i h, \mathbf{x}_i, z)$, implies that

$$\frac{dY_i}{dz} = \frac{\partial m_i}{\partial z} w_i h_i + m_i w_i \frac{dh_i}{dz} - m_i w_i \frac{dh_i}{dz} - \frac{\partial T_i}{\partial z} = \frac{\partial m_i}{\partial z} w_i h_i - \frac{\partial T_i}{\partial z}.$$

Substituting this relationship into the above expression for the change in hours worked gives

$$\frac{dh_i}{dz} = -\frac{\partial h_i}{\partial \left[(1-m_i) w_i\right]} w_i \frac{\partial m_i}{\partial z} + \frac{\partial h_i}{\partial Y_i} \left(\frac{\partial m_i}{\partial z} w_i h_i - \frac{\partial T_i}{\partial z} \right).$$

By using the definition of the uncompensated hours-of-work elasticity, $\varepsilon_i \equiv \frac{\partial h_i}{\partial[(1-m_i)w_i]} \frac{(1-m_i)w_i}{h_i}$, and the Slutsky equation (2.8), the above expression may be rewritten to

$$\frac{dh_i}{dz} = -\frac{h_i}{1-m_i} \left[\frac{\partial m_i}{\partial z} \varepsilon_i^c - \frac{1}{w_i h_i} \frac{\partial T_i}{\partial z} \theta_i \right]. \qquad (3.22)$$

Now, eq. (3.5) is obtained by inserting eqs (3.21) and (3.22) in eq. (3.2) and using eq. (2.6).

4. The Marginal Cost of Public Funds

In this section, we derive analytical measures of the marginal cost of funds (MCF), defined as the welfare cost from raising one additional Euro of government revenue. The approach is explained more carefully in Kleven and Kreiner (2006e).

Before proceeding with the derivations, we should emphasize a methodological choice which is being made. In the utility specification (2.1) we have not include public goods and, by implication, our results for the MCF will not incorporate any effects of public spending on labor supply and government revenue. This is the most common approach in the recent MCF literature (Dahlby, 1998; Sandmo, 1998), and it has been based on two different interpretations. In one interpretation – labelled the Stiglitz-Dasgupta-Atkinson-Stern approach by Ballard and Fullerton (1992) – public goods are simply assumed to be separable in utility so that spending has no effects on labor supply. In the other interpretation, any effects of spending on labor supply and revenue should be ascribed to the benefit side in the cost-benefit analysis used to evaluate a public project. As pointed out by Sandmo (1998), if the goal of MCF is to provide policy makers with a practical measure that can be applied across different public projects, it makes sense not to include the effects of government spending. Because if we did include such effects, each single public project would have to be assigned its own MCF.

The standard analytical approaches to measuring the MCF do not include distributional concerns. But in our framework where individuals are heterogeneous in terms of wages, preferences and fixed work costs, it is natural to start by considering a distributionally-weighted MCF, labelled the social marginal cost of public funds (SMCF) by Dahlby (1998). For this purpose, we assume an additive Bergson-Samuelson social welfare function $\Psi(\cdot)$. Aggregate welfare may then be written as

$$W = \sum_{i=1}^{I} \left[\int_{0}^{\bar{q}_i} \Psi_i \left(v_i \left(c_i, h_i \right) - q \right) p_i \left(q \right) dq + \int_{\bar{q}_i}^{\infty} \Psi_i \left(v_i \left(c_0, 0 \right) \right) p_i \left(q \right) dq \right] N_i, \quad (4.1)$$

where the first term reflects the contribution to welfare from those who are working, while the second term is the contribution to welfare from those outside the labor market. By considering a small change in the tax-transfer system – captured by a change in the z-parameter (dz) – we may define the social marginal cost of public funds in the following way

$$\text{SMCF} = -\frac{1}{\lambda} \frac{dW/dz}{dR/dz}, \quad (4.2)$$

where R denotes aggregate government revenue, and where we have defined

$$\lambda \equiv \sum_{i=1}^{I} \left[\int_{0}^{\infty} \Psi_i' \left(\cdot \right) \frac{\partial v_i \left(\cdot \right)}{\partial c} p_i \left(q \right) dq \right] N_i,$$

as the average social marginal utility of income for the entire population. The λ-parameter is of course necessary in the definition in order to convert the welfare effect in utils, dW, into a welfare effect in units of income.

In general, tax reforms affect labor supply along both the intensive and extensive margins. However, these labor supply adjustments have no first-order effects on aggregate welfare in (4.1), as long as we are considering marginal reforms. This is a result of the envelope theorem applied to each margin of response. Thus, because initial hours of work have been optimized by employed individuals (from eq. 2.4), small changes in hours worked have no effect on individual welfare v_i and therefore have no effect on aggregate welfare W. Likewise, changes in labor force participation – resulting from a changed threshold value $\bar{q}_i$ – will not affect aggregate welfare because marginal entrants were initially indifferent between working and not working (from eq. 2.5).

With these insights, it is straightforward to obtain the effect on aggregate welfare as

$$\frac{dW}{dz} = -\sum_{i=1}^{I}\left[\int_0^{\bar{q}_i} \Psi_i'(\cdot)\frac{\partial v_i(\cdot)}{\partial c_i}\frac{\partial T_i}{\partial z}p_i(q)\,dq + \int_{\bar{q}_i}^{\infty} \Psi_i'(\cdot)\frac{\partial v_i(\cdot)}{\partial c_i}\frac{\partial T_i^0}{\partial z}p_i(q)\,dq\right]N_i, \tag{4.3}$$

where $\partial T_i/\partial z$ denotes the mechanical increase in the tax payment for an employed group i individual (the increase in tax burden excluding any behavioral responses induced by the reform), while $\partial T_i^0/\partial z$ denotes the mechanical increase in the tax payment for an unemployed group i individual.

To simplify this expression, we define social welfare weights on working individuals in the following way

$$\begin{aligned} g_i(q) &\equiv \frac{1}{\lambda}\Psi_i'(v_i(c_i,h_i)-q)\frac{\partial v_i(c_i,h_i)}{\partial c_i}, \quad g_i \equiv \frac{\int_0^{\bar{q}_i} g_i(q)\,p_i(q)\,dq}{\int_0^{\bar{q}_i} p_i(q)\,dq}, \\ g_i^0 &\equiv \frac{1}{\lambda}\Psi_i'\left(v_i\left(c_i^0,h_i\right)\right)\frac{\partial v_i\left(c_i^0,0\right)}{\partial c_i^0} \end{aligned}$$

where $g_i(q)$ denotes the social marginal utility of income for a working individual in group i with fixed work cost q (relative to the average for the entire population), g_i is the average social marginal utility of income among the working population in group i, and g_i^0 is the social marginal utility of income for an unemployed individual in group i. Using these distributional weights, we may rewrite eq. (4.3) to

$$\frac{dW/dz}{\lambda} = -\sum_{i=1}^{I} \left[g_i \frac{\partial T_i}{\partial z} E_i + g_i^0 \frac{\partial T_i^0}{\partial z} (N_i - E_i) \right], \tag{4.4}$$

where $E_i = P_i(\bar{q}_i) N_i$ is the number of employed individuals in group i. This expression shows that the welfare effect is a weighted sum of the total mechanical increases in tax burdens for each group.

Besides the welfare effect just derived, the social marginal cost of public funds in (4.2) depends on the change in government revenue. Using the definitions in (3.2) and (3.7), SMCF may be written as

$$\text{SMCF} = \frac{-dW/dz}{\lambda \cdot dM/dz} \cdot \frac{1}{1-S}, \tag{4.5}$$

where dM/dz is the total mechanical effect of the reform on government revenue, while S is the degree of self-financing.

The marginal cost of public funds (MCF) focuses entirely on the efficiency aspect of taxation. This corresponds to assuming that the social value of an extra unit of consumption is uniform across all individuals in society ($g_i = g_i^0 = 1 \;\; \forall i$). In this case, it follows from (4.4) that $-\frac{1}{\lambda} \cdot dW/dz = dM/dz$ and (4.5) therefore simplifies to

$$\text{MCF} = \frac{1}{1-S}. \tag{4.6}$$

This expression reveals a simple relationship between the marginal cost of public funds and the degree of self-financing. For the six tax reforms considered in Section 3, we may obtain the MCF of the reforms simply by inserting the results for the degree of self-financing into the above expression.

5. The Trade-Off Between Efficiency and Equality

In Section 3 and Section 4, we studied policies that raise or reduce the aggregate tax revenue of the government. In this section, we study redistributive policies, defined as policy reforms that change the distribution of the tax burden between individuals in society without changing the aggregate tax revenue. The goal of the section is to derive analytical measures for the marginal trade-off between equity and efficiency. We will focus on the trade-off between equity and efficiency for two specific types of redistributional reforms: a traditional welfare policy that provides income support to all the poor and a working poor policy which target transfers to poor individuals conditional on working (in-work benefits). Our analysis generalizes the results in Immervoll *et al.* (2005), which assumed away income effects in labor supply decisions. In addition, we consider reforms that are targeted certain demographic groups.

Redistributive policies providing income support for the poor or the working poor come at the cost of reduced incomes and welfare among high-income earners. Let us consider a general small and revenue neutral tax reform dz. This reform creates losers and gainers. We denote by $dG \geq 0$ the aggregate welfare gains of those who gain from the reform and by $dL \leq 0$ the aggregate welfare change of those who loose from the reform. Note that in the case of a Pareto improving reform there are no losers and $dL = 0$.[1]

Due to behavioral responses to taxes and transfers, the decline in welfare for the rich may potentially be much higher than the welfare gain for the poor (i.e., $dG + dL < 0$), reflecting the distortionary effects of redistributive tax policy. A critical question then becomes how to evaluate the desirability of reforms involving such interpersonal utility trade-offs. The standard approach has been to specify a social welfare function involving certain welfare weights across individuals decreasing across the income distribution. Any given redistributive policy is then beneficial if it raises the value of the specified social welfare function. However, the interpersonal comparisons implied by the adopted welfare function are clearly subjective, and this limits the applicability of such an analysis as an input into the policy making process.

Following Browning and Johnson (1984), we divide the population into those who gain from the reform and those who lose from the reform. This partitioning of people will be endogenous both to the reform and to the behavioral responses created by the reform. This is an important point, which will be discussed later on when analyzing the distributional consequences of tax reforms. Within each of the two groups we assume that the social marginal utility of consumption is constant across individuals. We then define the interpersonal utility trade-off Ψ in the following way

$$\Psi = -\frac{dL}{dG}. \tag{5.1}$$

If the reform constitutes an increase in redistribution, Ψ gives the welfare cost to the rich from the transfer of one additional dollar of welfare to the poor (or the working poor). Conversely, if we are thinking about rolling back welfare programs,

[1] In contrast, if the reform is Pareto worsening, there are no gainers and $dG = 0$.

Ψ is the cost to the poor per dollar transferred to the rich. This interpersonal trade-off may be interpreted as a critical value for the relative social welfare weight between the two groups, i.e., the relative weight on those who gain such that the reform breaks even in terms of social welfare. The trade-off measure used here was originally proposed by Browning and Johnson (1984), and subsequently used by Ballard (1988), Triest (1994) and Immervoll *et al.* (2005).

The magnitude of Ψ reflects the degree to which there exists a trade-off between equity and efficiency. In the case with no behavioral responses to taxes and transfers, redistributive taxation does not imply lower efficiency, and there is no change in aggregate utilitarian welfare from the reform. Thus, the welfare gain of those who gain (the denominator) exactly equals the welfare loss of those who lose (the numerator), implying that Ψ is equal to one. Alternatively, a Ψ-value larger than one implies a trade-off between equity and efficiency (those who lose from the reform loose more than the gainers gain), whereas if Ψ is less than one there is no conflict between the two and the reform actually *increases* efficiency.

To derive Ψ for a general tax reform, we start by examining the impact on individual welfare from a marginal change in the reform parameter z. Let dW_i/dz denote the marginal welfare effect of an individual in group i measured in units of consumption. From eqs (2.1), (2.2), (2.4) and (2.5), we obtain[2]

$$\frac{dW_i}{dz} = \begin{cases} \frac{dv_i/dz}{\partial v_i/\partial c_i} = -\partial T_i/\partial z & q \leq q_i \\ \frac{dv_i/dz}{\partial v_i/\partial c_i^0} = -\partial T_i^0/\partial z & q > q_i \end{cases}, \tag{5.2}$$

where $T_i \equiv T(w_i l_i, \mathbf{x}_i, z)$ and $T_i^0 \equiv T(0, \mathbf{x}_i, z)$. The effect on individual welfare is

[2] In calculations of the *marginal* welfare effect, the choice of welfare measure does not matter. Thus, the welfare effect in equation (5.2) is consistent with all the common measures, for example the equivalent variation, the compensating variation, and the compensating surplus. See Fullerton (1991) for a discussion of these issues.

given simply by the direct (mechanical) change in the tax liability since, by the envelope theorem, a marginal tax-induced change in hours of work or participation does not affect utility as labour supply is initially at its optimal level.

Since the reform experiments which we consider do not take money away from those who are unemployed, i.e. $\partial T_i^0/\partial z \leq 0$, we may include these individuals among the gainers in the denominator of the Ψ-measure. Moreover, by defining G as the set of groups for which employed people gain from the reform, we may use eq. (5.2) to write Ψ in the following way

$$\Psi = -\frac{\sum_{i \notin G} \frac{\partial T_i}{\partial z} E_i}{\sum_{i \in G} \frac{\partial T_i}{\partial z} E_i + \frac{\partial T_i^0}{\partial z} (N - E)}, \tag{5.3}$$

where $E_i \equiv P_i(q_i) N_i$ denotes the number of employed people in group i, $E = \sum_i E_i$ is aggregate employment, and $N = \sum_i N_i$ is the total population.

Since we are considering redistributive policies, the tax reform is revenue neutral. It is central to note that this does not imply that the partial tax changes in the above expression sum to zero. Aggregating partial tax changes capture only the mechanical effect on government revenue, i.e., the effect in the absence of behavioral responses. The total effect on aggregate revenue of a reform dz is given by (3.5), which we repeat here

$$\begin{aligned} \frac{dR}{dz} &= \sum_{i=1}^{I} \left[\frac{\partial T_i}{\partial z} E_i + \frac{\partial T_i^0}{\partial z} (N_i - E_i) - \frac{m_i}{1 - m_i} \left(\frac{\partial m_i}{\partial z} w_i h_i \varepsilon_i^c - \frac{\partial T_i}{\partial z} \theta_i \right) E_i \right. \\ &\quad \left. - \frac{a_i + b_i}{1 - a_i - b_i} \left(\frac{\partial T_i}{\partial z} \eta_i - \frac{\partial T_i^0}{\partial z} \eta_i^0 \right) E_i \right]. \end{aligned} \tag{5.4}$$

We consider only policy reforms that change the distribution of the tax burden without changing the aggregate tax revenue, $dR/dz = 0$. For any of such reforms, we may calculate the equity-efficiency trade-off Ψ from formula (5.3). The first two terms in equation (5.4) are the mechanical effect (which we have previously denote by dM) of the tax reform. As we showed in (5.2), the mechanical effects are exactly equal to minus the aggregate welfare effect dW on the population. Let dB denote the behavioral effects, i.e., the third and fourth terms in equation (5.4). Equation (5.4) and revenue neutrality then imply that $dW = dG + dL = dB$. Hence, the aggregate change in welfare (adding the gains of gainers and the losses of losers) following the reform is exactly equal to the behavioral effect on government revenue. Thus, $-dB$ can be seen as the extra deadweight burden generated by the reform. Our equity-efficiency measure $\Psi = -dL/dG$ is larger than one if and only if $dB < 0$, i.e., the tax reform generates deadweight burden. For a given level of deadweight burden $-dB$, the larger the absolute value of gains and losses, the larger the amount of redistribution the reform achieves, and hence the smaller is Ψ.

In the following, we will concentrate on four simple tax reforms for which closed form expressions for Ψ may be obtained. These four types of policies are chosen so as to illuminate some of the most important trade-offs which policy makers are facing in connection with welfare reform.

5.1. Demogrant Policy

In this subsection, we analyze a small welfare reform which redistributes income from high-wage earners in the labour market to individuals earning low wages and to those who are not employed. In particular, the reform under consideration takes the form of a demogrant policy which raises the tax rate on all units of labour income by τ and returns the collected revenue as a lump sum TR to all individuals in the economy. This redistributive reform corresponds to an expansion of the traditional welfare programs financed by a general increase in tax rates.

The tax/transfer schedule is changed in the following manner:

$$\frac{\partial m_i}{\partial z} = \tau, \quad \frac{\partial T_i}{\partial z} = \tau w_i h_i - TR, \quad \frac{\partial T_i^0}{\partial z} = -TR. \tag{5.5}$$

Inserting these expressions in eq. (5.4) and setting dR/dz equal to zero, we obtain

$$TR \cdot N = \Omega_d \cdot \tau \sum_{i=1}^{I} w_i h_i E_i, \tag{5.6}$$

where

$$\Omega_d \equiv \frac{1 - S^E}{1 - S^A} \tag{5.7}$$

where S^A and S^E are defined in eqs (3.9) and (3.13). This expression shows that the aggregate lump sum transfer $TR \cdot N$ is equal to the direct increase in tax revenue from the imposition of τ multiplied by a factor Ω_d reflecting the behavioral responses to the reform. When $\Omega_d < 1$, a fraction $1 - \Omega_d$ of the mechanical tax revenue collections vanishes due to the behavioral responses to taxation, thereby reducing the amount of money which may be returned as a lump sum transfer.

Notice that this type of redistributive reform combines reform A and reform E studied in Section 3. The tax revenue is raised through a proportional tax increase. The term $1 - S^E$ in (5.7) gives the total increase in tax revenue in proportion to the mechanical increase in tax liabilities. The tax proceeds are paid out as lump sum transfers to everybody, and in order to pay out one euro we will need $1 - S^A$ euros because of the behavioral responses to the transfer payments.

In the special case of no labour supply responses along either the intensive or the extensive margins ($\varepsilon_i = \eta_i = 0$ for all i), there will be no behavioral revenue effects and Ω_d equals one. Likewise, if the initial tax system is a non-distortionary lump sum tax ($\tau_i = a_i = 0$ for all i), we get $\Omega_d = 1$.

Now, using eqs (5.5) and (5.6), we may rewrite (5.3) as

$$\Psi_d = 1 + \frac{1 - \Omega_d}{p_g \Omega_d - s_g} \geq 1, \tag{5.8}$$

where $p_g \equiv \left[\sum_{i \in G} E_i + (N - E)\right] / N$ denotes the population share for those who are gaining from the reform, while $s_g \equiv \sum_{i \in G} s_i$ is the cumulative wage share for those who are gaining. If we are considering a tax reform creating no efficiency loss ($\Omega_d = 1$), the interpersonal trade-off is exactly one, i.e., an additional euro transferred to the poor imposes a one-euro cost on the rich. However, if the redistributive reform generates an efficiency loss ($\Omega_d < 1$), and this is normally the case, it will cost more than one euro of welfare for the rich to transfer one euro to the poor.

5.2. Working Poor Policy

In this subsection, we compare the demogrant policy considered above with a reform which redistributes income to low-wage earners in the labour market, while keeping constant the income of those who are out of work. As before, the reform raises the tax rate on all units of labour income by τ, but now the collected revenue is returned only to those who are working positive hours. Conditional on labor force participation, the transfer is lump sum. This type of reform may be interpreted as the introduction of an Earned Income Tax Credit (EITC) financed by higher taxes on high-wage earners.

The tax/transfer schedule is changed in the following manner:

$$\frac{\partial m_i}{\partial z_i} = \tau, \quad \frac{\partial T_i}{\partial z} = \tau w_i l_i - TR, \quad \frac{\partial T_i^0}{\partial z} = 0. \tag{5.9}$$

Inserting these expressions in eq. (5.4) and setting dR/dz equal to zero, we obtain

$$TR \cdot E = \Omega_{wp} \cdot \tau \sum_{i=1}^{I} w_i h_i E_i, \tag{5.10}$$

where

$$\Omega_{wp} \equiv \frac{1 - S^E}{1 - S^B}, \tag{5.11}$$

where S^B and S^E are defined in eqs (3.10) and (3.13). As with the analogous eq. (5.6) for the demogrant policy, the above expression shows that the aggregate lump sum transfer, now $TR \cdot E$, is given by the direct revenue increase multiplied by a parameter Ω_{wp} capturing behavioral responses to the reform. The essential difference to the previous equation lies in the denominator of the Ω-parameter, which reflects the positive participation response arising because the transfer is given only to employed people, i.e. $S^B > S^A$. Because of these participation responses, the behavioral feed-back effects on revenue may be positive on net. Consequently, a redistribution towards the working poor may increase overall efficiency ($S^B > S^E$).

Inserting eqs (5.9) and (5.10) into (5.3), we get

$$\Psi_{wp} = 1 + \frac{1 - \Omega_{wp}}{e_g \Omega_{wp} - s_g}, \tag{5.12}$$

where $e_g \equiv \sum_{i \in G} e_i$ is the share of employed people gaining from the reform. In this expression, we have $\Psi_{wp} \gtreqless 1$ iff $\Omega_{wp} \gtreqless 1$. Hence, it is possible that the welfare cost to high-wage earners from the transfer of one dollar to low-wage earners is less than the dollar transferred. In this case there would be no conflict between equity and efficiency.

In the special case of no labour supply responses along the extensive margin ($\eta = 0$ and $\eta^0 = 0$) and no income effects in the hours-of-work decisions, the two types of tax reform which we have considered create identical behavioral responses. It is illuminating to compare our efficiency and trade-off measures Ω and Ψ in this special case.

Eqs (5.6) and (5.10) show immediately that $\Omega_d = \Omega_{wp}$, implying that the share of the projected mechanical increase in tax revenue which is lost through behavioral responses is the same for the two reforms. In other words, the additional deadweight burden, and hence the difference between gains dG and losses $-dL$, is the same for the two reforms. While the difference between gains and losses is identical, the absolute magnitudes tend to be higher in the case of a demogrant policy. In the demogrant policy, the unemployed obtain transfers without paying any taxes, whereas in the working poor policy everybody getting transfers also pays taxes. For this reason, the aggregate gain of the gainers dG and the aggregate loss of the losers $-dL$ will be higher for the demogrant policy. From the definition of the equity-efficiency trade-off in eq. (5.1), the larger magnitudes of both numerator and denominator (where the numerator is the larger number) implies that $\Psi_d < \Psi_{wp}$, i.e., the demogrant policy involves a more favorable trade-off than the in-work benefit reform. This result shows that, with no difference in the behavioral responses created by the reforms, the demogrant policy is "better" than the in-work benefits policy in the sense that it achieves more redistribution per euro of deadweight burden.

This difference in the trade-off for the two policies is part of a more general point. In general, the magnitude of Ψ depends on the earnings distribution among the people affected by the reform. Consider the working poor policy, for example. Since tax payments depend on earnings, if the distribution of earnings is initially relatively equal (workers are almost identical), the net mechanical tax change (equal to the welfare effect) will necessarily be almost the same for each individual (i.e., gains and losses are close to zero). In other words, with an equal earnings distribution, we get little redistribution, and for a given efficiency loss D, the trade-off measure Ψ becomes high. As the earnings distribution widens, gains and losses become bigger (more money is redistributed), and Ψ becomes lower. This implies that, for given labor supply elasticities, in-work benefits will be more desirable in countries with large earnings disparities.

5.3. Tagging of Transfers to Individuals with Certain Demographic Characteristics

In this subsection, we consider a target policy. Let Υ be the set of groups who are eligible for benefits. In this case, the tax/transfer schedule is changed in the following way:

$$\frac{\partial m_i}{\partial z_i} = \tau \; \forall i, \quad \frac{\partial T_i^0}{\partial z} = -TR \; \forall i \in \Upsilon, \quad \frac{\partial T_i^0}{\partial z} = 0 \; \forall i \notin \Upsilon, \tag{5.13}$$

$$\frac{\partial T_i}{\partial z} = \tau w_i l_i - TR \; \forall i \in \Upsilon, \quad \frac{\partial T_i}{\partial z} = \tau w_i l_i \; \forall i \notin \Upsilon. \tag{5.14}$$

Only the target groups receive the transfer TR while all employed individuals pay higher taxes in order to finance the outlays (the term $\tau w_i l_i$).

By inserting the above expressions in eq. (5.4) and setting dR/dz equal to zero, we obtain

$$TR \cdot \sum_{\Upsilon} N_i = \Omega_t \cdot \tau \sum_{i=1}^{I} w_i h_i E_i, \tag{5.15}$$

where

$$\Omega_t \equiv \frac{1 - S^E}{1 - S^C},$$

where S^C and S^E are defined in eqs (3.11) and (3.13).

The trade-off between efficiency and equality, Ψ_t, may be calculated using eqs (5.13), (5.14) and (5.15) into (5.3).

5.4. Targeted Working Poor Policy

In this subsection, we consider a working poor policies where the in-work benefits are targeted to individuals with certain demographic characteristics (e.g. single parents). Let Υ be the set of groups eligible for in-work benefits. Then, the tax/transfer schedule is changed in the following way:

$$\frac{\partial \tau_i}{\partial z_i} = \tau \ \forall i, \quad \frac{\partial T_i^0}{\partial z} = 0 \ \forall i, \tag{5.16}$$

$$\frac{\partial T_i}{\partial z} = \tau w_i l_i - TR \ \forall i \in \Upsilon, \quad \frac{\partial T_i}{\partial z} = \tau w_i l_i \ \forall i \notin \Upsilon. \tag{5.17}$$

The impact of the targeting is seen in the last equation. The in-work benefits, TR, are only paid out to individuals belonging to the target groups Υ while all employed individuals pay higher taxes in order to finance the outlay (the term $\tau w_i l_i$).

By inserting the above expressions in eq. (5.4) and setting dR/dz equal to zero, we obtain

$$TR \cdot \sum_{\Upsilon} E_i = \Omega_{wpt} \cdot \tau \sum_{i=1}^{I} w_i h_i E_i, \tag{5.18}$$

where

$$\Omega_{wpt} \equiv \frac{1 - S^E}{1 - S^D}, \tag{5.19}$$

where S^C and S^D are defined in eqs (3.12) and (3.13).

The trade-off between efficiency and equality, Ψ_{wpt}, may be calculated using eqs (5.16), (5.17) and (5.18) into (5.3).

6. Conditions Under Which Tax Cuts Pay for Themselves

This section derives conditions under which tax cuts pay for themselves. We will first analyze, for given tax rates, how large elasticities need to be in order to make tax cuts completely self-financed. Second, we analyze, for given labor supply elasticities, how large initial tax rates need to be in order to make tax cuts self-financed. That is, we solve for the tax rates that maximizes government revenue (also called Laffer tax rates). Of course, the calculations will depend on the type of reform considered. We will for this exercise focus on Reform G. For this reform, tax cuts pay for themselves when the degree of self-financing in eq. (3.18) equals one. This will be the starting point for the calculations below.

6.1. Required Elasticities

6.1.1. The Compensated Hours-of-Work Elasticity

In this section, we derive the level of the *average* compensated hours-of-work elasticity $\bar{\varepsilon}^c$ required to make Reform G exactly self-financed ($S_k^G = 1$) for given values of other parameters. The hours-of-work elasticities may still vary according to earnings and demographic groups. We will assume that the compensated hours-of-work elasticity for group (i, j) equals

$$\varepsilon_{ij}^c = \gamma_{ij}\bar{\varepsilon}^c,$$

where the γ-parameters fulfill

$$\sum_{i=k}^{100} \sum_{j=1}^{J} \gamma_{ij} \frac{E_{ij}}{E_i} = 1.$$

This ensures that the average hours-of-work elasticity equals $\bar{\varepsilon}^c$. By inserting this in (3.18) and setting S_{k}^G equal to one, we obtain

$$\bar{\varepsilon}^c = \frac{1 + \sum_{i=k}^{100} s_i B_i - \sum_{i=k}^{100} s_i C_i}{\sum_{i=k}^{100} \Phi_i s_i A_i},$$

where

$$A_i \equiv \sum_{j=1}^{J} \frac{m_{ij}}{1 - m_{ij}} \gamma_{ij} \frac{E_{ij}}{E_i},$$

$$B_i \equiv \sum_{j=1}^{J} \frac{m_{ij}}{1 - m_{ij}} \theta_{ij} \frac{E_{ij}}{E_i},$$

$$C_i \equiv \sum_{j=1}^{J} \frac{a_{ij} + b_{ij}}{1 - a_{ij} - b_{ij}} \eta_{ij} \frac{E_{ij}}{E_i}.$$

It is possible to increase government revenue through tax cuts if the average hours-of-work elasticity is larger than $\bar{\varepsilon}^c$.

6.1.2. The Participation Elasticity

In this section, we derive the level of the average participation elasticity $\bar{\eta}$ that is sufficient to make Reform G self-financed for given values of other parameters. The participation elasticities may still vary according to earnings and demographic groups. We will assume that the participation elasticity for group (i, j) equals

$$\eta_{ij} = \gamma_{ij} \bar{\eta},$$

where the γ-parameters fulfill

$$\sum_{i=k}^{100} \sum_{j=1}^{J} \gamma_{ij} \frac{E_{ij}}{E_i} = 1,$$

which ensures that the average participation elasticity is in fact equal to $\bar{\eta}$. By inserting the above relationship in eq. (3.18) and setting S_{k}^{G} equal to one, we obtain

$$\bar{\eta} = \frac{1 - \sum_{i=k}^{100} \Phi_i s_i A_i + \sum_{i=k}^{100} s_i B_i}{\sum_{i=k}^{100} s_i C_i},$$

where

$$\begin{aligned} A_i &\equiv \sum_{j=1}^{J} \frac{m_{ij}}{1-m_{ij}} \varepsilon_{ij}^{c} \frac{E_{ij}}{E_i}, \\ B_i &\equiv \sum_{j=1}^{J} \frac{m_{ij}}{1-m_{ij}} \theta_{ij} \frac{E_{ij}}{E_i}, \\ C_i &\equiv \sum_{j=1}^{J} \frac{a_{ij}+b_{ij}}{1-a_{ij}-b_{ij}} \gamma_{ij} \frac{E_{ij}}{E_i}. \end{aligned}$$

6.2. Required Tax Rates

In this section, we analyze, for given labor supply elasticities, how large initial tax rates need to be in order to make tax cuts self-financed.

6.2.1. Only Hours-of-Work Responses

We first focus on a special case without participation responses ($\eta_{ij} = 0$ for all deciles $i \geq k$). Moreover, we assume that the effective marginal tax rate is constant from percentile k and onwards. Let m denote this tax rate. In the following, we analyze how large the pre-reform effective marginal tax rate needs to be in order to make Reform G — a small reduction in m — self-financed.

With a constant marginal tax rate, m, from percentile k and onwards, eq.

(3.18) gives

$$
\begin{aligned}
S_{\mathrm{k}}^{G} &= \frac{m}{1-m}\sum_{i=k}^{100}\frac{w_i h_i}{\sum_{i=k}^{100}(w_i h_i - y_k)}\sum_{j=1}^{J}\varepsilon_{ij}^{c}\frac{E_{ij}}{E_i} \\
&\quad -\frac{m}{1-m}\sum_{i=k}^{100}\frac{w_i h_i - y_k}{\sum_{i=k}^{100}(w_i h_i - y_k)}\sum_{j=1}^{J}\theta_{ij}\frac{E_{ij}}{E_i}.
\end{aligned}
$$

By setting S_{k}^{G} equal to one, we obtain the Laffer tax rate

$$
m = \frac{1}{1+\sum_{i=k}^{100}\left(\alpha_k^i \varepsilon_i^c - \beta_k^i \theta_i\right)} \tag{6.1}
$$

where $\alpha_k^i \equiv \frac{w_i h_i}{\sum_{j=k}^{100}(w_j h_j - y_k)}$, $\beta_k^i \equiv \frac{w_i h_i - y_k}{\sum_{j=k}^{100}(w_j h_j - y_k)}$, $\varepsilon_i = \sum_{j=1}^{42}\varepsilon_{ij}^c \frac{N_{ij}}{N_i}$ and $\theta_i = \sum_{j=1}^{42}\theta_{ij}\frac{N_{ij}}{N_i}$.

In the special case, with constant hours-of-work elasticities at all deciles above k, the expression (6.1) simplifies to

$$
m = \frac{1}{1+\varepsilon^c \alpha_k - \theta}\ \forall i \geq k, \tag{6.2}
$$

where $\alpha_k \equiv \frac{\sum_{i=k}^{100} w_i h_i}{\sum_{i=k}^{100}(w_i h_i - y_k)}$. Notice, that in this special case, the size of m is governed completely by the shape of the earnings distribution, reflected in the α_k-parameter. Empirical evidence shows that α_k is increasing in the income percentile k but that it converges towards some constant (see Saez, 2004 and Kleven and Kreiner, 2006c). This implies that the shape of the wage distribution makes the size of the Laffer rate a decreasing function of the income threshold above which taxes are changed.

6.2.2. Only Participation Responses

We now focus on the opposite special case, i.e. a situation without labor supply responses along the intensive margin ($\varepsilon_{ij}^c = \theta_{ij} = 0$ for all deciles $i \geq k$). Moreover, we assume that the effective participation tax rate is constant from percentile k

and onwards. Let $a+b$ denote this tax rate. In the following, we analyze how large the pre-reform effective participation tax rate needs to be in order to make Reform G — a small reduction in $a+b$ — self-financed. In this case expression (3.18) simplifies to

$$S_{\mathrm{k}}^{G}=\frac{a+b}{1-a-b}\sum_{i=k}^{100}\beta_{k}^{i}\eta_{i},$$

where $\eta_i \equiv \sum_{j=1}^{J}\eta_{ij}\frac{E_{ij}}{E_i}$ and $\beta_k^i \equiv \frac{w_i h_i - y_k}{\sum_{i=k}^{100}(w_i h_i - y_k)}$. By setting S_{k}^{G} equal to one, we obtain the Laffer tax rate

$$a+b=\frac{1}{1+\sum_{i=k}^{100}\beta_{k}^{i}\eta_{i}}. \tag{6.3}$$

Notice, that if the participation elasticities are all equal to some given level η then the revenue-maximizing tax rate is constant at all income thresholds k:

$$a+b=\frac{1}{1+\eta}.$$

Constant participation elasticities are, however, not realistic. Empirical evidence shows that low-income individuals have high elasticities while high-income individuals have very small elasticities. This implies from formula (2.6) that the revenue-maximizing tax rate is an increasing function of the percentile k from which the the tax rate is changed.

6.2.3. Combining Hours-of-Work Responses and Participation Responses

In this section, we include both intensive and extensive labor supply responses. However, in order to solve for a Laffer tax rate in this case, it is necessary to work with a slightly different definition of the participation elasticity. Let $\tilde{\eta}_{ij}$ be the percentage change in participation caused by a one percent change in the wage rate, $\frac{\partial P_i}{\partial w_i}\frac{w_i}{P_i}$. This elasticity is related to the participation elasticity in (2.9) according to

$$\eta_{ij} = \frac{1 - a_{ij} - b_{ij}}{1 - m_{ij}}\tilde{\eta}_{ij}.$$

We may now rewrite (3.18) as

$$\begin{aligned} S_{\mathrm{k}}^{G} &= \sum_{i=k}^{100} \Phi_i s_i A_i - \sum_{i=k}^{100} s_i B_i + \sum_{i=k}^{100} s_i C_i \\ A_i &\equiv \sum_{j=1}^{J} \frac{m_{ij}}{1 - m_{ij}} \varepsilon_{ij}^{c} \frac{E_{ij}}{E_i}, \\ B_i &\equiv \sum_{j=1}^{J} \frac{m_{ij}}{1 - m_{ij}} \theta_{ij} \frac{E_{ij}}{E_i}, \\ C_i &\equiv \sum_{j=1}^{J} \frac{a_{ij} + b_{ij}}{1 - m_{ij}} \tilde{\eta}_{ij} \frac{E_{ij}}{E_i}, \end{aligned}$$

where $s_i = \frac{w_i h_i - y_k}{\sum_{i=k}^{100}(w_i h_i - y_k)}$ and $\Phi_i = \frac{w_i h_i}{w_i h_i - y_k}$.

We assume that the effective marginal tax rate is constant from percentile k and onwards. Let m denote this tax rate. In the following, we analyze how large the pre-reform effective marginal tax rate needs to be in order to make Reform G — a small reduction in m — self-financed. In this case, the above expression becomes

$$S_{\mathrm{k}}^{G} = \frac{m}{1 - m} \sum_{i=k}^{100} \left(\alpha_k^i \varepsilon_i^c - \beta_k^i \theta_i \right) + \sum_{i=k}^{100} s_i \sum_{j=1}^{J} \frac{a_{ij} + b_{ij}}{1 - m} \tilde{\eta}_{ij} \frac{E_{ij}}{E_i}.$$

Adjustments in the marginal tax rate m influence the participation tax rate according to

$$a_{ij} + b_{ij} = (m - \bar{m}_{ij}) \frac{w_i h_i - y_k}{w_i h_i} + \bar{a}_{ij} + \bar{b}_{ij},$$

where $\bar{m}_{ij}$ and $\bar{a}_{ij} + \bar{b}_{ij}$ are the tax rates individuals face in the existing system according to the data. By combining the two above expressions, we arrive at

$$S_{\text{k}}^{G} = \frac{m}{1-m}\sum_{i=k}^{100}\left(\alpha_k^i \varepsilon_i^c - \beta_k^i \theta_i\right) + \sum_{i=k}^{100} s_i \sum_{j=1}^{J} \frac{(m-\bar{m}_{ij})\frac{w_i h_i - y_k}{w_i h_i} + \bar{a}_{ij} + \bar{b}_{ij}}{1-m} \tilde{\eta}_{ij} \frac{E_{ij}}{E_i}.$$

The Laffer tax rate is found by setting S_{k}^{G} equal to one and isolating m. This gives

$$m = \frac{1 - \sum_{i=k}^{100} \beta_k^i \left(\sum_{j=1}^{J}\left(\bar{a}_{ij} + \bar{b}_{ij}\right)\tilde{\eta}_{ij}\frac{E_{ij}}{E_i} - \frac{w_i h_i - y_k}{w_i h_i}\sum_{j=1}^{J}\bar{m}_{ij}\tilde{\eta}_{ij}\frac{E_{ij}}{E_i}\right)}{1 + \sum_{i=k}^{100}\left(\alpha_k^i \varepsilon_i^c - \beta_k^i \theta_i\right) + \sum_{i=k}^{100}\beta_k^i \frac{w_i h_i - y_k}{w_i h_i}\sum_{j=1}^{J}\tilde{\eta}_{ij}\frac{E_{ij}}{E_i}}.$$

Our numerical analysis of Laffer tax rates is based on this expression. To better understand the effects underlying the above result, it is useful to look at the special case where hours-of-work elasticities (ε^c and θ) are constant and the participation elasticities and tax rates do not depend on j. In this situation, the above formula simplifies to

$$m = \frac{1 - \sum_{i=k}^{100} \beta_k^i \left(\bar{a}_i + \bar{b}_i - \bar{m}_i \frac{w_i h_i - y_k}{w_i h_i}\right)\tilde{\eta}_i}{1 + \alpha_k \varepsilon^c - \theta + \sum_{i=k}^{100} \beta_k^i \frac{w_i h_i - y_k}{w_i h_i}\tilde{\eta}_i}, \tag{6.4}$$

where $\alpha_k \equiv \frac{\sum_{i=k}^{100} w_i h_i}{\sum_{i=k}^{100}(w_i h_i - y_k)}$. This expression may be compared to the result without participation responses in eq. (6.2). The only difference is the last term in the numerator and the last term in the denominator of the above expression. These terms arise because of participation responses which reduce the Laffer tax rate. More interestingly, the above expression shows that the profile of the participation elasticities are important. Empirical evidence indicates that the participation elasticities decrease with earnings. This implies, ceteris paribus, that the revenue-maximizing tax rate m is an increasing function of the percentile k from which the the tax rate is changed. Hence, expression (6.4) combines the effects described in the previous two sections.

Part II

Numerical Analysis for Denmark

7. Simulation Methodology

The numerical results reported in Kleven and Kreiner (2006c,d) are calculated from the formulas in Part I using a microsimulation approach. The computations require information on earnings, effective tax rates, and labor supply elasticities. Information on earnings and effective tax rates are obtained from individual data using the Danish Law Model while labor supply elasticities are set in accordance with the empirical labor supply literature. Kleven and Kreiner (2006b,c) provide a short survey of this literature and describe the labor supply elasticity constellations applied in the numerical analysis.

The theoretical analysis in Part I is based on a discrete formulation dividing the population into a number of distinct subgroups. In the empirical application, we have to define these subgroups. Here it is important to choose a level of disaggregation which adequately captures the observed heterogeneity in the sample. Because tax rates, wage income and (potentially) labor supply elasticities are strongly heterogeneous *and* correlated across individuals, one could make substantial errors by aggregating too much. Our simulations will be based on a disaggregation into 100 earnings deciles where each decile is divided into 42 subgroups depending on demographic characteristics (see Table 1 for details on the demographic groups). This gives 4,200 groups in total. For each group, we calculate an earnings level, a marginal tax rate, and a participation tax rate for

TABEL 1
Demografiske grupper

Gruppe	Demografiske karakteristika
1	Dansk baggrund, 18-25 år
2	Dansk baggrund, 25-54 år, Enlig, Ingen hjemmeboende børn
3	Dansk baggrund, 25-54 år, Enlig, Hjemmeboende børn
4	Dansk baggrund, 25-54 år, Mand, Arbejdende ægtefælle, Ingen hjemmeboende børn
5	Dansk baggrund, 25-54 år, Kvinde, Arbejdende ægtefælle, Ingen hjemmeboende børn
6	Dansk baggrund, 25-54 år, Mand, Arbejdende ægtefælle, Hjemmeboende børn
7	Dansk baggrund, 25-54 år, Kvinde, Arbejdende ægtefælle, Hjemmeboende børn
8	Dansk baggrund, 25-54 år, Mand, Ikke-arbejdende ægtefælle, Ingen hjemmeboende børn
9	Dansk baggrund, 25-54 år, Kvinde, Ikke-arb. ægtefælle, Ingen hjemmeboende børn
10	Dansk baggrund, 25-54 år, Mand, Ikke-arbejdende ægtefælle, Hjemmeboende børn
11	Dansk baggrund, 25-54 år, Kvinde, Ikke-arbejdende ægtefælle, Hjemmeboende børn
12	Dansk baggrund, 55-65 år, Enlig
13	Dansk baggrund, 55-65 år, Gift, Mand
14	Dansk baggrund, 55-65 år, Gift, Kvinde
15	Vestlig indvandrerbaggrund, 18-25 år
16	Vestlig indvandrerbaggrund, 25-54 år, Enlig, Ingen hjemmeboende børn
17	Vestlig indvandrerbaggrund, 25-54 år, Enlig, Hjemmeboende børn
18	Vestlig indvandrerbaggrund, 25-54 år, Mand, Arbejdende ægtefælle, Ingen hjemmeboende børn
19	Vestlig indvandrerbaggrund, 25-54 år, Kvinde, Arbejdende ægtefælle, Ingen hjemmeboende børn
20	Vestlig indvandrerbaggrund, 25-54 år, Mand, Arbejdende ægtefælle, Hjemmeboende børn
21	Vestlig indvandrerbaggrund, 25-54 år, Kvinde, Arbejdende ægtefælle, Hjemmeboende børn
22	Vestlig indvandrerbaggrund, 25-54 år, Mand, Ikke-arb. ægtefælle, Ingen hjemmeboende børn
23	Vestlig indvandrerbaggrund, 25-54 år, Kvinde, Ikke-arb. ægtefælle, Ingen hjemmeboende børn
24	Vestlig indvandrerbaggrund, 25-54 år, Mand, Ikke-arbejdende ægtefælle, Hjemmeboende børn
25	Vestlig indvandrerbaggrund, 25-54 år, Kvinde, Ikke-arbejdende ægtefælle, Hjemmeboende børn
26	Vestlig indvandrerbaggrund, 55-65 år, Enlig
27	Vestlig indvandrerbaggrund, 55-65 år, Gift, Mand
28	Vestlig indvandrerbaggrund, 55-65 år, Gift, Kvinde
29	Ikke vestlig indvandrerbaggrund, 18-25 år
30	Ikke vestlig indvandrerbaggrund, 25-54 år, Enlig, Ingen hjemmeboende børn
31	Ikke vestlig indvandrerbaggrund, 25-54 år, Enlig, Hjemmeboende børn
32	Ikke vestlig indvandrerbaggrund, 25-54 år, Mand, Arbejdende ægtefælle, Ingen hjemmeboende børn
33	Ikke vestlig indvandrerbaggrund, 25-54 år, Kvinde, Arbejdende ægtefælle, Ingen hjemmeboende børn
34	Ikke vestlig indvandrerbaggrund, 25-54 år, Mand, Arbejdende ægtefælle, Hjemmeboende børn
35	Ikke vestlig indvandrerbaggrund, 25-54 år, Kvinde, Arbejdende ægtefælle, Hjemmeboende børn
36	Ikke vestlig indvandrerbaggrund, 25-54 år, Mand, Ikke-arb. ægtefælle, Ingen hjemmeboende børn
37	Ikke vestlig indvandrerbaggrund, 25-54 år, Kvinde, Ikke-arb. ægtefælle, Ingen hjemmeboende børn
38	Ikke vestlig indvandrerbaggrund, 25-54 år, Mand, Ikke-arbejdende ægtefælle, Hjemmeboende børn
39	Ikke vestlig indvandrerbaggrund, 25-54 år, Kvinde, Ikke-arbejdende ægtefælle, Hjemmeboende børn
40	Ikke vestlig indvandrerbaggrund, 55-65 år, Enlig
41	Ikke vestlig indvandrerbaggrund, 55-65 år, Gift, Mand
42	Ikke vestlig indvandrerbaggrund, 55-65 år, Gift, Kvinde

a representative individual by taking the average of the individuals in the group. The data on earnings and the calculation of effective tax rates are described in the next sections.

Since the theoretical results in Part I are based on a small reform methodology, the computed effects represent first-order approximations to the true effects. The big advantage of the small reform methodology is its simplicity and transparency. To evaluate the reforms, we need only to estimate earnings and tax rates and to set elasticities based on the empirical literature. Moreover, we do not need to specify utility functions or estimate (or calibrate) utility parameters in order to calculate welfare effects.[1] Neither do we need to estimate fixed costs of working. It is, however, important to keep in mind that the results only apply to small changes in the existing system.

[1] This is because the welfare effect of tax reform reflects the externalities created by changes in government revenue, a results that follows from the definition of excess burden and the application of envelope theorems.

8. Data on Taxes, Benefits, and Earnings

Information on earnings and effective tax rates are obtained from the Danish Law Model. The Law Model consists of two main components: (*i*) A database containing information on individuals in Denmark and (*ii*) the rules in the Danish tax-benefit system converted into computer language (a detailed description is provided in Finansministeriet (2003)). The primary statistical data in the database is collected from a 33.3 per cent random sample of the Danish population. We further restrict the sample to individuals in the age 18 to 65 who are working full-time or part-time (excluding students) and who are registered as having positive ATP-contributions and earnings above 35,600 Danish kroner. For a few of the computations, we also use the Law Model to obtain the number of individuals outside the labor market in the different demographic groups.

The model population is from year 2003 but earnings, tax payments etc. are adjusted to 2006 price- and wage levels. The database contains a considerable number of items of information on each person, and by combining this information with the computer algorithms representing the existing tax and benefit legislation, the model is able to compute a range of tax and benefit amounts for each observation unit in the database.

The computer algorithms captures the full range of institutional features of tax and benefit systems in the year 2006. This includes detailed income definitions (such as taxable income or "means" relevant for computing income-tested benefits), precise definitions of family and assessment units (such as who counts as a "child" for the purpose of particular tax or benefit rules), thresholds, floors, ceilings and relevant tax rates as well as specific eligibility rules, claw-back rates or income disregards used in computing benefit entitlements. The considerable level of detail makes it possible to derive a finely grained picture of tax burdens and benefit entitlements and how these vary with earnings and individual or family characteristics. Moreover, it is possible to calculate the change in the net tax-benefit position of an individual if, say, the earnings of the individual change. This feature is crucial in the calculation of the effective tax rates. Since the Law Model takes into account interactions between different policy instruments (such as the taxation of benefits) and household members' incomes (e.g. that social assistance is a function of family rather than individual income) we are able to capture all relevant effects on total household income of an earnings change for a particular household member. A detailed description of the Danish tax-benefit system and the way it is implemented into the Law Model may be found in Chapter 6 and Chapter 7 of Finansministeriet (2004).

Yearly pre-tax earnings are calculated directly from the data. In the calculations, we include pension contributions by both the employees and the employers. The data on earnings are used as input in the formulas and are also used to divide the individuals into earnings deciles.

9. Calculation of Effective Tax Rates

The effective marginal tax rate of an individual is computed by increasing the yearly earnings y of the individual by $1,000$ kroner. The Law Model computes all the changes in taxes and benefits of the household, and provides thereby an estimate of the new net-tax burden of the household, $T(y+1,000)$. The effective marginal tax rate may then be calculated as

$$\hat{m} = \frac{T(y+1,000) - T(y)}{1,000},$$

where $T(y)$ is the net-tax burden of the household before the increase in earnings. In this calculation, we have disregarded earnings-related pension contributions. In order to account for the fact that a part of an increase in earnings normally comes in the form of a higher pension, we use instead the formula

$$\tilde{m} = \frac{T(y+1,000) - T(y) + 1,000 \cdot p \cdot \tau_p}{1,000 \cdot (1+p)},$$

where p denotes total pension contributions in proportion of pre-pension earnings, while τ_p is the effective marginal tax rate on the retirement benefit when the individual is retired. In the calculations, we set τ_p equal to 55 per cent as also done by the Ministry of Finance in similar calculations.

The purchasing power of labor income is also influenced by the substantial tax on consumption in Denmark. Let τ_c be the tax rate on individual consumption, i.e. the amount of consumption taxes in proportion to the pre-tax value of consumption. Then the effective marginal tax rate on earnings becomes

$$m = \frac{\tilde{m} + \tau_c}{1 + \tau_c}.$$

The Value Added Tax alone is 25 per cent. On top of that comes excise taxes on specific goods such as cars, gasoline, alcohol, and tobacco. This difference in consumption taxes implies that the effective marginal tax rate depends on the consumption behavior of the individual. Unfortunately, it is impossible to account for such differences in consumption behavior. The calculation of the consumption tax rate is based on the methodology of Mendoza *et al.* (1994) which calculates an (average) consumption tax rate, τ_c, from National Accounts. In our calculations of the effective tax rates, we use a Ministry of Finance estimate of τ_c equal to 33 per cent.

In order to compute the participation tax rate, we first compute the difference between current household taxes and benefits and household taxes and benefits when the earnings of the individual are set to zero: $T(y) - T(0)$. We then divide this difference by earnings y in order to obtain the participation tax rate (as in the theoretical part, the participation rate may be written as an average tax rate a plus a benefit rate b), i.e.

$$\hat{a} + \hat{b} = \frac{T(y) - T(0)}{y}.$$

When accounting for the effect of earnings-related pensions, the participation rate becomes

$$\tilde{a} + \tilde{b} = \frac{T(y) + y \cdot p \cdot \tau_p - T(0)}{y \cdot (1 + p)}.$$

Finally, we also need to include consumption taxes in the calculation of effective tax rates. This gives

$$a + b = \frac{\tilde{a} + \tilde{b} + \tau_c}{1 + \tau_c}.$$

Kleven and Kreiner (2006c) reports effective marginal tax rates and participation tax rates for the Danish population.

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